

Pair Production of Arbitrary Spin Particles by Electromagnetic Fields

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Abstract

The exact solutions of the wave equation for arbitrary spin particles in the field of the soliton-like electric impulse were obtained. The differential probability of pair production of particles by electromagnetic fields has been evaluated on the basis of the exact solutions. As a particular case, the particle pair producing in the constant and uniform electric field were studied.

1 Introduction

One of the most interesting nonlinear phenomena in quantum field theory is the e^+e^- -pair production by a static and uniform electric field [1]. It is impossible to describe this effect in the framework of the perturbation theory which is nonperturbative effect occurring in strong external fields. With the aid of the semiclassical imaginary-time method, which is valid only for weak electromagnetic fields, the pair production probability of arbitrary spin particles was studied in [2] for the case of constant and uniform fields. We calculated the probability of pair production of arbitrary spin particles with the electric dipole moments (EDM) and anomalous magnetic moments (AMM) by constant and uniform electromagnetic fields of arbitrary strength [3]. Spontaneous e^+e^- -pair creation requires the strong constant and uniform electric field of the value (critical field) $E_c \simeq 1.3 \times 10^{18} \text{ V/m}$ that is difficult to achieve experimentally. In the proposed X-ray free electron laser facilities at SLAC and DESY [4], [5] the strength of the field can reach $E \simeq 0.1E_c$. It is well known, however, that one laser beam, which is described by a plane electromagnetic wave, can not produce pairs of particles [1]. With the help of two coherent laser beams, it possible to form a standing wave which produces pairs. Such a possibility of e^+e^- -pair creation by X-ray free electron laser is discussed in [5], [6], [7]. The radiation field of two crossing laser beams can

be approximated by spatial uniform and time varying electric field [6]. If the laser pulse duration decreases the probability of electron-positron creation increases sharply [7]. Therefore, it is interesting to investigate the general case of the pair production of arbitrary spin particles by the electric impulse of the electromagnetic field.

The paper is organized as follows. In Section 2, the formulation of arbitrary spin particles on the basis of the Lorentz representation $(s, 0) \oplus (s - 1/2, 1/2) \oplus (0, s) \oplus (1/2, s - 1/2)$ (\oplus means the direct sum) in the form of the first order relativistic wave equation is briefly considered. The spin operator, which commutes with the equation matrix, is defined. The exact solutions of the wave equation and the differential probability of pair production of arbitrary spin particles in the field of the soliton-like electric impulse are investigated in Section 3. In Section 4, the case of the particle pair producing in the constant and uniform electric field is studied. We discuss the results obtained in Section 5.

We use units in which $\hbar = c = 1$, and four-vectors are defined as $V_\mu = (\mathbf{V}, V_4)$, $\mathbf{V} = (V_m)$ ($m = 1, 2, 3$), $V_4 = iV_0$. The scalar product is $\mathbf{V}\mathbf{P} = V_m P_m$. The Greek and Latin letters run numbers 1, 2, 3, 4, and 1, 2, 3, respectively. A summation is implied over the repeated indices.

2 Arbitrary spin particles

We consider the first order relativistic wave equation (RWE) for arbitrary spin particles suggested in [8]. In an external electromagnetic field this RWE takes the form

$$(\beta_\mu \mathcal{D}_\mu + m) \phi_\epsilon(x) = 0, \quad (1)$$

where $\mathcal{D}_\mu = \partial_\mu - ieA_\mu$, $\partial_\mu = (\partial/\partial m, \partial/i\partial t)$, the t is the time, $A_\mu = (\mathbf{A}, iA_0)$ is four-vector-potential. The matrices and the wave function of RWE (1) are given by (in our notation)

$$\beta_m = i\epsilon g \begin{pmatrix} 0 & S_m & -K_m^+ \\ -S_m & 0 & 0 \\ K_m & 0 & 0 \end{pmatrix}, \quad \beta_4 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2)$$

$$\phi_\epsilon(x) = \begin{pmatrix} \psi_\epsilon(x) \\ \chi_\epsilon(x) \\ \Omega_\epsilon(x) \end{pmatrix},$$

where g is a gyromagnetic ratio, $g = 1/s$ (s is a spin of particles), K_m^+ is Hermitian-conjugate matrix, and the parameter $\epsilon = 1$ corresponds to the $(s, 0) \oplus (s - 1/2, 1/2)$, and $\epsilon = -1$ to the $(0, s) \oplus (1/2, s - 1/2)$ representations of the Lorentz group. Thus, the magnetic moment of arbitrary spin particles is $\mu = \mu_B = e/(2m)$, μ_B is the Bohr magneton. The square $(2s+1) \times (2s+1)$ -dimensional spin matrices S_m and rectangular $(2s-1) \times (2s+1)$ -dimensional matrices K_m obey the relationships [8]

$$\begin{aligned} [S_i, S_j] &= i\epsilon_{ijk}S_k, \quad (S_1)^2 + (S_2)^2 + (S_3)^2 = s(s+1), \\ S_i S_j + K_i^+ K_j &= is\epsilon_{ijk}S_k + s^2\delta_{ij} \quad (i, j, k = 1, 2, 3), \end{aligned} \quad (3)$$

where ϵ_{ijk} is antisymmetric tensor, $\epsilon_{123} = 1$. The functions $\psi_\epsilon(x)$, $\chi_\epsilon(x)$ possess $2s+1$ components, and the $\Omega_\epsilon(x)$ has $2s-1$ components, so that the wave function $\phi_\epsilon(x)$ possesses $6s+1$ components. The system of two equations (1) for $\epsilon = 1$ (the wave function $\phi_+(x)$) and $\epsilon = -1$ (the wave function $\phi_-(x)$) is a parity invariant because at the P -transformation, the representation of the Lorentz group $(s, 0) \oplus (s - 1/2, 1/2)$ is transformed into $(0, s) \oplus (1/2, s - 1/2)$ representation. The whole wave function $\phi(x) = \phi_+(x) \oplus \phi_-(x)$ has $2(6s+1)$ components.

Here we do not discuss the case of arbitrary EDM and AMM of spin particles that requires the consideration of high dimension RWE of the first order (see [9]).

Equation (1) is form-invariant under the Lorentz transformation due to the relation [8]

$$[M_{\mu\nu}, \beta_\sigma] = \delta_{\nu\sigma}\beta_\mu - \delta_{\mu\sigma}\beta_\nu, \quad (4)$$

where $M_{\mu\nu}$ are generators of the Lorentz group, and the commutator is defined as usual: $[A, B] = AB - BA$.

Now we construct the spin operator of a particle in external electromagnetic field. In this paper we investigate particles in electromagnetic fields that are spatially homogenies and dependent only on time. Therefore, consider the electromagnetic vector-potential of the form $A_\mu = A_\mu(t)$. The solution of Eq. (1) can be represented as

$$\phi_\epsilon(x) = \Phi_\epsilon(t) \exp(i\mathbf{p}\mathbf{x}), \quad (5)$$

where \mathbf{p} is the momentum vector and the function $\Phi(t)$ obeys the equation

$$[i\beta_m(p_m - eA_m(t)) - i\beta_4(\partial_t + eA_4(t)) + m] \Phi_\epsilon(t) = 0, \quad (6)$$

where $\partial_t = \partial/\partial t$.

The spin operator of particles corresponding to the time-dependent vector-potential may be defined in the form

$$\hat{S}_{\mathbf{p}} = \frac{\mathbf{J}(\mathbf{p} - e\mathbf{A}(t))}{|\mathbf{p} - e\mathbf{A}(t)|}, \quad J_m = \frac{1}{2}\epsilon_{mnk}M_{nk} = \begin{pmatrix} S_m & 0 & 0 \\ 0 & S_m & 0 \\ 0 & 0 & \Sigma_m \end{pmatrix}, \quad (7)$$

where Σ_m are generators of SU(2) group in the (2s-1) - representation [8]. We consider the case when the electric field, \mathbf{E} , vanishes at $t \rightarrow \pm\infty$. It is easy to verify with the help of Eq. (4) that the spin operator $\hat{S}_{\mathbf{p}}$ (7) commutes with the matrix of Eq. (6) for asymptotic states:

$$[\hat{S}_{\mathbf{p}}, \Lambda] = 0 \quad (t \rightarrow \pm\infty), \quad (8)$$

$$\Lambda \equiv i\beta_m(p_m - eA_m(t)) - i\beta_4(\partial_t + eA_4(t)) + m.$$

Then the wave function $\Phi_\epsilon(t)$ (and $\phi_\epsilon(x)$) is eigenfunction of the spin operator for asymptotic states:

$$\hat{S}_{\mathbf{p}}\Phi_\epsilon(t) = s_z\Phi_\epsilon(t) \quad t \rightarrow \pm\infty, \quad (9)$$

where s_z is a spin projection, $s_z = -s, \dots, s$.

The second order wave equation for the function $\psi_\epsilon(x)$ follows from Eq. (1), and is given by [8]:

$$\left[\mathcal{D}_\mu^2 - m^2 - \frac{e}{2}gF_{\mu\nu}\Sigma_{\mu\nu}^{(\epsilon)} \right] \psi_\epsilon(x) = 0, \quad (10)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the strength tensor of electromagnetic fields. Eq. (10) may be considered as the arbitrary - spin generalization of the Feynman-Gell-Mann [10] equation (see also [11]). The generators $\Sigma_{\mu\nu}^{(\epsilon)}$ of the representations $(s, 0)$ for $\epsilon = 1$ and $(0, s)$ for $\epsilon = -1$ are

$$\Sigma_{ij}^{(\epsilon)} = \epsilon_{ijk}S_k, \quad \Sigma_{0k}^{(\epsilon)} = -i\epsilon S_k. \quad (11)$$

The components $\chi_\epsilon(x)$, $\Omega_\epsilon(x)$ of the whole wave function ϕ_ϵ , (2), read

$$\chi_\epsilon(x) = \frac{1}{m} [(i\partial_t - eA_0) + i\epsilon g(\mathbf{SD})] \psi_\epsilon(x), \quad \Omega_\epsilon(x) = -\frac{\epsilon g}{m}(\mathbf{KD}) \psi_\epsilon(x). \quad (12)$$

So, if one finds a solution $\psi_\epsilon(x)$ of the second order Eq. (10), the functions $\chi_\epsilon(x)$, $\Omega_\epsilon(x)$ can be obtained from Eqs. (12).

In order to define a relativistically invariant bilinear form, we introduce the $(12s + 2)$ -dimensional wave function [8]

$$\phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_-(x) \end{pmatrix}. \quad (13)$$

Then the Lorentz-invariant bilinear form is given by [8]

$$\bar{\phi}(x)\phi(x) = \phi^+(x)\eta\phi(x) = \phi_+^+(x)\phi_-(x) + \phi_-^+(x)\phi_+(x), \quad \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (14)$$

where $\phi^+(x)$ is the Hermitian-conjugate wave function. From Eqs. (5),(7), (9), (12) one may find the relation for the asymptotic state ($t \rightarrow \pm\infty$)

$$\begin{aligned} \phi_+^+(x)\phi_-(x) &= \psi_+^+(t) \left[1 - \frac{1}{m^2} \left(i\overleftarrow{\partial}_t + eA_0(t) \right) (i\partial_t - eA_0(t)) \right. \\ &\quad \left. - i\frac{g s_z}{m^2} \left(\overleftarrow{\partial}_t |\mathbf{p} - e\mathbf{A}(t)| + |\mathbf{p} - e\mathbf{A}(t)| \partial_t \right) - \frac{(\mathbf{p} - e\mathbf{A}(t))^2}{m^2} \right] \psi_-(t), \end{aligned} \quad (15)$$

where the derivative $\overleftarrow{\partial}_t$ acts on the left standing function. The second term in Eq. (14), $\phi_-^+(x)\phi_+(x) = \left(\phi_+^+(x)\phi_-(x) \right)^+$, can be obtained with the help of the complex conjugation of expression (15). Eqs. (14), (15) can be used for the normalization of the wave function.

3 The soliton-like electric impulse

Let us consider Eq. (6) for a particle in the field of the electric impulse field. The non-stationary spatially homogeneous electric field is defined by the 4-vector potential

$$A_\mu = (0, 0, A_3(t), 0), \quad A_3(t) = -a_0 \tanh k_0 t, \quad (16)$$

so that the electric field represents the soliton-like electric impulse:

$$E_3 = \frac{a_0 k_0}{\cosh^2 k_0 t}, \quad (17)$$

$\mathbf{E} = (0, 0, E_3)$. The problem of the pair production of particles with the spin 1/2 by the field (17) was considered in [12], [13]. Here we generalize this

result on the case of arbitrary spin particles. From Eq. (10) with the help of Eqs. (16)-(17), we arrive at the equation

$$\left[\frac{d^2}{dt^2} + p_0^2 + 2ep_3a_0 \tanh k_0t + e^2a_0^2 \tanh^2 k_0t + \frac{ie\epsilon ga_0k_0}{\cosh^2 k_0t} S_3 \right] \Psi_\epsilon(t) = 0, \quad (18)$$

where $p_0^2 = m^2 + \mathbf{p}^2$,

$$\psi_\epsilon(x) = \Psi_\epsilon(t) \exp(i\mathbf{p} \cdot \mathbf{x}).$$

The equations (18) with the representations $\epsilon = 1$ and $\epsilon = -1$ are connected by the complex conjugation.

We note that when the transverse momentum of particles approaches to zero, $p_\perp \rightarrow 0$ ($p_\perp^2 = p_1^2 + p_2^2$), the spin operator (7), for the field (16), becomes $\hat{S}_\mathbf{p} = J_3$. Then according to Eq. (9) the wave function $\Psi_\epsilon(t)$ obeys the equation

$$S_3 \Psi_\epsilon(t) = s_z \Psi_\epsilon(t). \quad (19)$$

Taking into consideration Eq. (19), one of solutions to Eq. (18) is given by

$${}_-\Psi_\epsilon^{(s_z)}(t) = {}_-\mathcal{N}(-z)^{i\mu} (1-z)^\lambda F(i\mu-i\nu+\lambda, i\mu+i\nu+\lambda; 1+2i\mu; z) \Psi_0^{(s_z)}, \quad (20)$$

where $F(a, b; c; z)$ is the hypergeometric function [14], $\Psi_0^{(s_z)}$ is the constant normalized eigenvector obeying Eq. (19), ${}_-\mathcal{N}$ is the normalization constant, and parameters μ, ν, λ, z are

$$\begin{aligned} z &= \exp(2k_0t), \quad 2k_0\mu = \sqrt{p_\perp^2 + m^2 + (p_3 - ea_0)^2} \\ 2k_0\nu &= \sqrt{p_\perp^2 + m^2 + (p_3 + ea_0)^2}, \\ \lambda &= \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{e^2a_0^2}{k_0^2} + i\epsilon s_z g \frac{ea_0}{k_0}}. \end{aligned} \quad (21)$$

Solution (20) has the following asymptotic limit at $t \rightarrow \pm\infty$:

$$\lim_{t \rightarrow +\infty} {}_-\Psi_\epsilon^{(s_z)}(t) = {}_-\mathcal{N} \left[C_1^{(s_z)} \exp(2i\nu k_0t) + C_2^{(s_z)} \exp(-2\nu k_0t) \right] \Psi_0^{(s_z)}, \quad (22)$$

$$\lim_{t \rightarrow -\infty} {}_-\Psi_\epsilon^{(s_z)}(t) = {}_-N \exp(2i\mu k_0 t) \Psi_0^{(s_z)},$$

where coefficients $C_1^{(s_z)}$ and $C_2^{(s_z)}$ are given by

$$\begin{aligned} C_1^{(s_z)} &= \frac{\Gamma(2i\nu)\Gamma(1+2i\mu)}{\Gamma(i\mu+i\nu+\lambda)\Gamma(1+i\mu+i\nu-\lambda)}, \\ C_2^{(s_z)} &= \frac{\Gamma(-2i\nu)\Gamma(1+2i\mu)}{\Gamma(i\mu-i\nu+\lambda)\Gamma(1+i\mu-i\nu-\lambda)}, \end{aligned} \quad (23)$$

and $\Gamma(x)$ is the Gamma-function. We imply here that the transverse momentum of particles approaches to zero, $p_\perp \rightarrow 0$. Equation (22) indicates that the solution ${}_-\Psi_\epsilon^{(s_z)}(t)$ corresponds to the negative frequency at $t \rightarrow -\infty$ (that is why we use the subscript $-$ in the wave function ${}_-\Psi_\epsilon^{(s_z)}(t)$). It also follows from Eqs. (22) that $2k_0\mu$ and $2k_0\nu$ are the kinetic energies of particles at $t \rightarrow -\infty$ and $t \rightarrow +\infty$, respectively [12], and the function ${}_-\Psi_\epsilon^{(s_z)}(t)$ contains only negative frequency at $t \rightarrow -\infty$. The solution ${}_+\Psi_\epsilon^{(s_z)}(t)$ to Eq. (18) with the positive frequency at $t \rightarrow -\infty$ may be obtained from Eq. (20) by the substitution $\mu \rightarrow -\mu$:

$${}_+\Psi_\epsilon^{(s_z)}(t) = {}_+N(-z)^{-i\mu} (1-z)^\lambda F(-i\mu-i\nu+\lambda, -i\mu+i\nu+\lambda; 1-2i\mu; z) \Psi_0^{(s_z)}, \quad (24)$$

so that

$$\lim_{t \rightarrow -\infty} {}_+\Psi_\epsilon^{(s_z)}(t) = {}_+N \exp(-2i\mu k_0 t) \Psi_0^{(s_z)}. \quad (25)$$

Eq. (18) is invariant, as well as equation for spin 1/2 [12], under the replacement: $t \rightarrow -t$, $e \rightarrow -e$, $s_z \rightarrow -s_z$ ($\mu \leftrightarrow \nu$, $\lambda \rightarrow \lambda$). Making this substitution in Eq. (20), we find the solution to Eq. (18) with the negative frequency at $t \rightarrow \infty$:

$$\begin{aligned} {}_-\Psi_\epsilon^{(s_z)}(t) &= {}_-N(-z)^{i\nu} (1-z^{-1})^\lambda F(i\nu-i\mu+\lambda, i\nu+i\mu+\lambda; 1+2i\nu; z^{-1}) \Psi_0^{(s_z)}, \\ \lim_{t \rightarrow \infty} {}_-\Psi_\epsilon^{(s_z)}(t) &= {}_-N \exp(2i\nu k_0 t) \Psi_0^{(s_z)}. \end{aligned} \quad (26)$$

In the same manner, using the replacement $\nu \rightarrow -\nu$, we obtain from Eq. (26) the solution with the positive frequency

$$\begin{aligned} {}_+\Psi_\epsilon^{(s_z)}(t) &= {}_+N(-z)^{-i\nu} (1-z^{-1})^\lambda F(-i\nu-i\mu+\lambda, -i\nu+i\mu+\lambda; 1-2i\nu; z^{-1}) \Psi_0^{(s_z)}, \\ \lim_{t \rightarrow \infty} {}_+\Psi_\epsilon^{(s_z)}(t) &= {}_+N \exp(-2i\nu k_0 t) \Psi_0^{(s_z)}. \end{aligned} \quad (27)$$

With the help of Eqs. (22), (27) one can obtain the asymptotic (at $t \rightarrow \infty$) relation

$$_-\Psi_\epsilon^{(s_z)}(\infty) = a^{(s_z)} - \Psi_\epsilon^{(s_z)}(\infty) + b^{(s_z)} + \Psi_\epsilon^{(s_z)}(\infty), \quad (28)$$

where

$$a^{(s_z)} = \frac{-N}{-N} C_1^{(s_z)}, \quad b^{(s_z)} = \frac{-N}{+N} C_2^{(s_z)}. \quad (29)$$

It follows from Eqs. (2), (5), (12), (28) that the wave function of the first order RWE (1) obeys at $t \rightarrow \infty$ the same equation as Eq. (28):

$$_-\phi_\epsilon^{(s_z)}(x) = a^{(s_z)} - \phi_\epsilon^{(s_z)}(x) + b^{(s_z)} + \phi_\epsilon^{(s_z)}(x), \quad (30)$$

where functions $_-\phi_\epsilon^{(s_z)}(x)$ (with the negative frequency at $t \rightarrow -\infty$), $_-\phi_\epsilon^{(s_z)}(x)$ (with the negative frequency at $t \rightarrow \infty$), $+\phi_\epsilon^{(s_z)}(x)$ (with the positive frequency at $t \rightarrow \infty$) satisfy Eq. (9).

The pair production probability of particles by electromagnetic fields can be obtained through the asymptotic of the exact solutions of wave equations [15]. Relation (30) and complex conjugated equation generate the Bogolyubov transformations of creation and annihilation operators. Coefficients $a^{(s_z)}$ and $b^{(s_z)}$, Eq. (29), contain information about pair creation and obey the relations

$$\begin{aligned} |a^{(s_z)}|^2 - |b^{(s_z)}|^2 &= 1 \quad \text{for bosons,} \\ |a^{(s_z)}|^2 + |b^{(s_z)}|^2 &= 1 \quad \text{for fermions.} \end{aligned} \quad (31)$$

According to the approach [15], the density of pairs of arbitrary spin particles created during all time is given by

$$n^{(s)} = \sum_{s_z} \int |b^{(s_z)}|^2 \frac{d^3p}{(2\pi)^3}. \quad (32)$$

The formula (32) generalizes the result [15] on the case of arbitrary spin particles. The difference of Eqs. (23), (32) from expressions for spin zero and one half [15] is in the parameter λ , Eq. (21) and values $a^{(s_z)}$ and $b^{(s_z)}$.

3.1 Spin-0, and 1/2 particles

Let us consider a particular case of spinless particles, $s = 0$. One may get the Klein – Gordon – Fock equation by setting $s_z = 0$ in Eq. (18). According to

Eq. (15), normalization constants in Eq. (29) depend on the spin of particles. For spin-zero, one has to use the normalization constants [15]:

$${}_+N^{(0)} = {}_-N^{(0)} = (4k_0\mu)^{-1/2}, \quad {}_+N^{(0)} = {}_-N^{(0)} = (4k_0\nu)^{-1/2}. \quad (33)$$

Putting $s_z = 0$ in Eqs. (21), (23), (29), and using the formulas [14]:

$$|\Gamma(1+iy)|^2 = \frac{\pi y}{\sinh \pi y}, \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}, \quad (34)$$

we obtain

$$\begin{aligned} \lambda &= \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{e^2 a_0^2}{k_0^2}}, \\ |a^{(0)}|^2 &= \frac{\sin \pi(i\mu + i\nu + \lambda) \sin \pi(-i\mu - i\nu + \lambda)}{\sinh 2\pi\mu \sinh 2\pi\nu}, \\ |b^{(0)}|^2 &= \frac{\sin \pi(i\mu - i\nu + \lambda) \sin \pi(-i\mu + i\nu + \lambda)}{\sinh 2\pi\mu \sinh 2\pi\nu}. \end{aligned} \quad (35)$$

The Bogolubov coefficients $a^{(0)}$, $b^{(0)}$, Eq. (35), obey the relation (31) for bosons. With the help of Eqs. (32), (35) one can get the known result [15] (see also [16]):

$$n^{(0)} = \int \frac{\cos^2 \pi\vartheta' + \sinh^2 \pi(\mu - \nu)}{\sinh 2\pi\mu \sinh 2\pi\nu} \frac{d^3 p}{(2\pi)^3}, \quad \vartheta' = \sqrt{\frac{1}{4} - \frac{e^2 a_0^2}{k_0^2}}. \quad (36)$$

Consider the case of spin-1/2 particles. Putting $s_z = 1/2$, $g = 2$, we find from Eqs. (21), (23), (34)

$$\begin{aligned} \lambda &= 1 + i\vartheta, \quad \vartheta = \frac{ea_0}{k_0}, \\ |C_1^{(1/2)}|^2 &= \frac{\mu(\mu + \nu - \vartheta) \sinh \pi(\mu + \nu + \vartheta) \sinh \pi(\mu + \nu - \vartheta)}{\nu(\mu + \nu + \vartheta) \sinh 2\pi\mu \sinh 2\pi\nu}, \\ |C_2^{(1/2)}|^2 &= \frac{\mu(\mu - \nu - \vartheta) \sinh \pi(\mu - \nu + \vartheta) \sinh \pi(\mu - \nu - \vartheta)}{\nu(\mu - \nu + \vartheta) \sinh 2\pi\mu \sinh 2\pi\nu}. \end{aligned} \quad (37)$$

Equations (37) correspond to the spin projection $s_z = 1/2$ and the representation with the parameter $\epsilon = 1$. For the spin projection $s_z = -1/2$ one

should make the replacement $\vartheta \rightarrow -\vartheta$ in Eqs. (37). Using the normalization constants [15]

$$\begin{aligned} {}_+N^{(1/2)} &= [8k_0\mu (2k_0\mu - p_3 + ea_0)]^{-1/2}, \\ {}_-N^{(1/2)} &= [8k_0\mu (2k_0\mu + p_3 - ea_0)]^{-1/2}, \\ {}_+N^{(1/2)} &= [8k_0\nu (2k_0\nu - p_3 - ea_0)]^{-1/2}, \\ {}_-N^{(1/2)} &= [8k_0\nu (2k_0\nu + p_3 + ea_0)]^{-1/2}, \end{aligned} \quad (38)$$

and the relations

$$\frac{2k_0\nu - p_3 - ea_0}{2k_0\mu + p_3 - ea_0} = \frac{\mu - \nu + \vartheta}{\nu - \mu + \vartheta}, \quad \frac{2k_0\nu + p_3 + ea_0}{2k_0\mu + p_3 - ea_0} = \frac{\mu + \nu + \vartheta}{\nu + \mu - \vartheta}, \quad (39)$$

one may find the coefficients

$$\begin{aligned} a^{(1/2)} &= \frac{\sinh \pi(\mu + \nu + \vartheta) \sinh \pi(\mu + \nu - \vartheta)}{\sinh 2\pi\mu \sinh 2\pi\nu}, \\ b^{(1/2)} &= \frac{\sinh \pi(\nu - \mu - \vartheta) \sinh \pi(\mu - \nu - \vartheta)}{\sinh 2\pi\mu \sinh 2\pi\nu}. \end{aligned} \quad (40)$$

Coefficients $a^{(1/2)}$, $b^{(1/2)}$, Eq. (40), satisfy Eq. (31) for fermions. We recover from Eqs. (29), (32), (40) the density of electron-positron pairs created by the electric field (17) [15]:

$$n^{(1/2)} = 2 \int \frac{\sinh \pi(\nu - \mu - \vartheta) \sinh \pi(\mu - \nu - \vartheta)}{\sinh 2\pi\mu \sinh 2\pi\nu} \frac{d^3p}{(2\pi)^3}. \quad (41)$$

The coefficient 2 in Eq. (41) is due to two spin pojections $s_z = \pm 1/2$.

4 Constant and uniform electric field

We can get the approximation of a constant field by considering the limiting case $ea_0/m \rightarrow \infty$ [15], [16]. Setting $a_0 = E/k_0$, one arrives from Eq. (16) at $k_0 \rightarrow 0$:

$$A_3 = -\frac{E}{k_0} \tanh k_0 t \rightarrow -Et. \quad (42)$$

The vector-potential (42) describes the uniform and constant electric field. From Eq. (21) we find the approximate relations

$$\mu \simeq \nu \simeq \frac{eE}{2k_0^2} + \frac{p_\perp^2 + m^2}{4eE},$$

$$\lambda \simeq \frac{1}{2}(1 + s_z g) + i \frac{eE}{k_0^2}. \quad (43)$$

With the help of asymptotic formula [14]

$$|\Gamma(x + iy)| = \sqrt{2\pi}|y|^{x-1/2} \exp\left(-\frac{\pi|y|}{2}\right) \quad \text{at } |y| \rightarrow \infty, \quad (44)$$

we find the approximate relations

$$\begin{aligned} |\Gamma(i\mu + i\nu + \lambda)|^2 &= 2\pi \left(\frac{eE}{k_0^2}\right)^{s_z g} \exp\left(-\frac{\pi eE}{k_0^2}\right), \\ |\Gamma(1 + i\mu + i\nu - \lambda)|^2 &= 2\pi \left(\frac{eE}{k_0^2}\right)^{-s_z g} \exp\left(-\frac{\pi eE}{k_0^2}\right). \end{aligned} \quad (45)$$

Using Eqs. (34), (45) it is not difficult to obtain from Eq. (23) the expression

$$|C_2^{(s_z)}|^2 = \exp\left(-\frac{\pi(p_\perp^2 + m^2)}{eE}\right). \quad (46)$$

For the approximation considered the Bogolyubov coefficient (29) $b^{(s_z)} = C_2^{(s_z)}$. As a result, from Eq. (32) one obtains the density of arbitrary spin particles pairs created during all time

$$n^{(s)} = \sum_{s_z} \int \exp\left[-\frac{\pi(p_\perp^2 + m^2)}{eE}\right] \frac{d^3 p}{(2\pi)^3}. \quad (47)$$

For the constant and uniform electric field, according to the approach [15], one should make the replacement

$$\int dp_3 \rightarrow eET, \quad (48)$$

where T is infinite time of observation. Inserting the replacement (48) into Eq. (47) and calculating integral, we obtain the intensity of pair production of arbitrary spin particles by the constant and uniform electric field

$$I(E) = \frac{n^{(s)}}{T} = (2s + 1) \frac{e^2 E^2}{8\pi^3} \exp\left(-\frac{\pi m^2}{eE}\right). \quad (49)$$

Expression (48) is in agreement with the differential probability calculated on the basis of exact solutions of equations for arbitrary spin particles in the

external constant and uniform electromagnetic fields [3]. The function (49) possesses the nonanalytical dependence on the electric field, which indicates that the intensity (49) can not be received by perturbative theory.

The intensity of pair production of arbitrary spin particles, Eq. (49), is $(2s + 1)$ times that for the scalar particle intensity due to the $(2s + 1)$ physical degrees of freedom of the arbitrary spin field. The probability of pair production decreases rapidly with the mass of particles because of the exponential factor.

5 Conclusion

The number of pairs of arbitrary spin particles produced by a uniform soliton-like electric field was obtained on the basis of exact solutions. The intensity of pair production was expressed through the Bogolyubov coefficient C_2 which with the coefficient C_1 relate the asymptotics of exact solutions at $t \rightarrow \pm\infty$. At a particular case ($k_0 \rightarrow 0$), when the field is converted into constant electric field, we come to our earlier result. This case corresponds to the adiabatic regime [7] which can be now achieved experimentally. In another case, with the small pulse duration, the probability can increase considerably.

For spin 0 and 1/2, the probabilities of pair production reduce to expressions found in [15]. Although the light electron-positron pairs are created by the smallest value of the electric field, the general case of arbitrary spin particles is of definite theoretical interest.

Pair production of particles are described by nonperturbative effects of electrodynamics and it is impossible to obtain them by summing of definite terms of perturbation theory. Therefore, the observation of the pair production by the external electromagnetic field experimentally is of great scientific interest.

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